ENTRANCE EXAMS FOR THE I.B. DIPLOMA PROGRAM

INDICATIVE PAST PAPERS

1. A. Answer if the following statements are True (T) or False (F):

I. $(x-y)(-x+y)=x^2-y^2$

II. $(-x-2y)^2 = x^2+4xy+4y^2$

B. Fill in the blanks:

I. $(\ldots-\ldots)^2 = 9x^2 \cdots 6xy \cdots \ldots$

II. $(x-3)^3 = \ldots$

C. Factorize the following expressions completely:

I. $x^2y-x^2-xy+x+y-1=$

II. $16x^2+40xy+25y^2 =$

III. $16y^2-9(x+y)^2 =$

IV. $2x^2-3x+1=$

2. A. Factorize completely the following expressions:

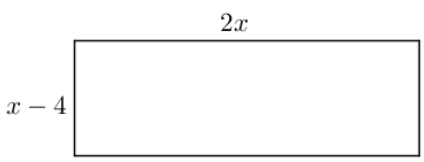
I. $4\alpha^2 b - \frac{b^3}{9}$

II. $5\alpha^n - 20\alpha^{n+1}b + 20\alpha^{n+2}b^2, n \in \mathbb{N}.$

III. $3x(1-y)^2 - 6x^2(y-1)^2 - 3x(1-y)$

B. The numbers $\alpha = \frac{2}{3+\sqrt{5}}$ and $b = \frac{2}{3-\sqrt{5}}$ are given.
Find the numerical value of the expression $\alpha + b$.

3. The surface of a carpet is shown below. The dimensions of the carpet are in meters.



I. Write down an expression for the area A, in m^2, of the carpet.
If the area of the carpet is 10m^2, then:

II. Calculate the value of x.

III. Hence, write down the value of the length and of width of the carpet, in meters.

4. Given the equation: $2x^2 + 5x - 1 = 0$,

I. Show that it has two real and distinct solutions, x_1, x_2.

II. Find the value of the following expressions: $x_1 + x_2$, $x_1 \cdot x_2$ and $\frac{1}{x_1} + \frac{1}{x_2}$.

III. Construct a quadratic equation with roots: $r_1 = \frac{1}{x_1}$ and $r_2 = \frac{1}{x_2}$.

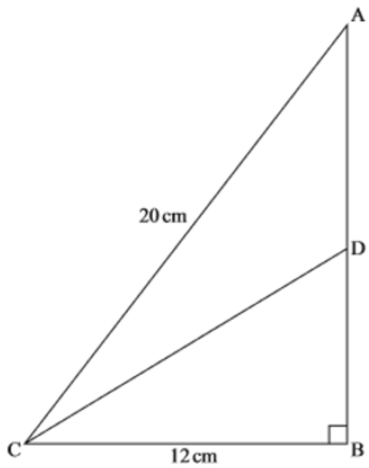
5. In triangle ABC, AC=20cm, BC=12 and $A\hat{B}C = 90^\circ$.

I. Find the length ofAB.

D is the point on AB such that $\tan(D\hat{C}B) = 0{,}6$.

II. Find the length ofDB.

III. Find the area of the triangle ADC.



6. **1.** Solve the following inequality and write down its solutions in the form of an interval Δ.

$$\frac{|2x-1|}{3} - 1 < \frac{3 - |1-2x|}{4}$$

2. If $x \in \Delta$, show that the following expression A is constant (independent of x), where

$$A = \frac{\sqrt{x^2+2x+1}}{x+1} + \frac{\sqrt{x^2-4x+4}}{x-2}$$

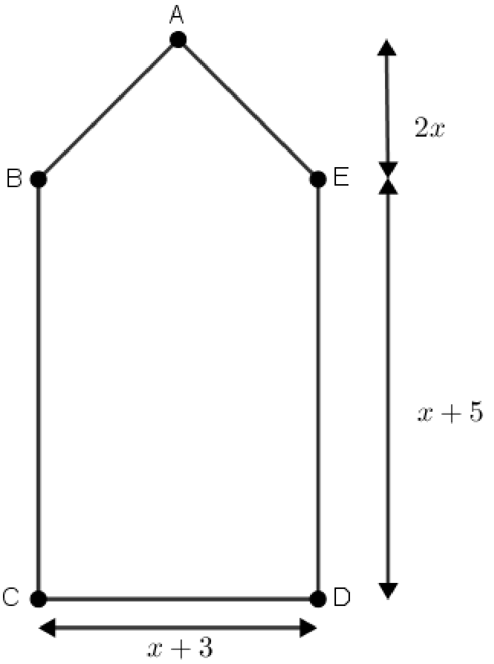
7. The base of an electric iron has the shape of a pentagon ABCDE as shown below where BCDE is a parallelogram with sides $(x+3)$cm and $(x+5)$cm and ABE is an isosceles triangle (AB=AE) with height $2x$ cm. The area of ABCDE is 21 cm^2.

IV. Express the area of ABCDE in terms of x.

V. Show that $2x^2+11x-6=0$.

VI. Find the length of CD.

VII. Find approximately the angle $B\hat{A}E$.



Trigonometric Ratio Table

angle	sin	cos	tan	angle	sin	cos	tan
1°	0,0175	0,9998	0,0175	46°	0.7193	0,6947	1,036
2°	0,0349	0,9994	0,0349	47°	0.7314	0,6820	1,072
3°	0,0523	0,9986	0,0524	48°	0.7431	0,6691	1,111
4°	0,0698	0,9976	0,0699	49°	0.7547	0,6561	1,150
5°	0,0872	0,9962	0,0875	50°	0.7660	0,6428	1,192
6°	0,1045	0,9945	0,1051	51°	0.7771	0,6293	1,235
7°	0,1219	0,9925	0,1228	52°	0.7880	0,6157	1,280
8°	0,1392	0,9903	0,1405	53°	0.7986	0,6018	1,327
9°	0,1564	0,9877	0,1584	54°	0,8090	0,5878	1,376
10°	0,1736	0,9848	0,1763	55°	0,8192	0,5736	1,428
11°	0,1908	0,9816	0,1944	56°	0,8290	0,5592	1,483
12°	0,2079	0,9781	0,2126	57°	0,8387	0,5446	1,540
13°	0,2250	0,9744	0,2309	58°	0,8480	0,5299	1,600
14°	0,2419	0,9703	0,2493	59°	0,8572	0,5150	1,664
15°	0,2588	0,9659	0,2679	60°	0,8660	0,5000	1,732
16°	0,2756	0,9613	0,2867	61°	0,8746	0,4848	1,804
17°	0,2924	0,9563	0,3057	62°	0,8829	0,4695	1,881
18°	0,3090	0,9511	0,3249	63°	0,8910	0,4540	1,963
19°	0,3256	0,9455	0,3443	64°	0,8988	0,4384	2,050
20°	0,3420	0,9397	0,3640	65°	0,9063	0,4226	2,145
21°	0,3584	0,9336	0,3839	66°	0,9135	0,4067	2,246
22°	0,3746	0,9272	0,4040	67°	0,9205	0,3907	2,356
23°	0,3907	0,9205	0,4245	68°	0,9272	0,3746	2,475
24°	0,4067	0,9135	0,4452	69°	0,9336	0,3584	2,605
25°	0,4226	0,9063	0,4663	70°	0,9397	0,3420	2,747
26°	0,4384	0,8988	0,4877	71°	0,9455	0,3256	2,904
27°	0,4540	0,8910	0,5095	72°	0,9511	0,3090	3,078
28°	0,4695	0,8829	0,5317	73°	0,9563	0,2924	3,271
29°	0,4848	0,8746	0,5543	74°	0,9613	0,2756	3,487
30°	0,5000	0,8660	0,5774	75°	0,9659	0,2586	3,732
31°	0,5150	0,8572	0,6009	76°	0,9703	0,2419	4,011
32°	0,5299	0,8480	0,6249	77°	0,9744	0,2250	4,332
33°	0,5446	0,8387	0,6494	78°	0,9781	0,2079	4,705
34°	0,5592	0,8290	0,6745	79°	0,9816	0,1908	5,145
35°	0,5736	0,8192	0,7002	80°	0,9848	0,1736	5,671
36°	0,5878	0,8090	0,7265	81°	0,9877	0,1564	6,314
37°	0,6018	0,7986	0,7536	82°	0,9903	0,1392	7,115
38°	0,6157	0,7880	0,7813	83°	0,9925	0,1219	8,144
39°	0,6293	0,7771	0,8098	84°	0,9945	0,1045	9,514
40°	0,6428	0,7660	0,8391	85°	0,9962	0,0872	11,43
41°	0,6561	0,7547	0,8693	86°	0,9976	0,0698	14,30
42°	0,6691	0,7431	0,9004	87°	0,9986	0,0523	19,08
43°	0,6820	0,7314	0,9325	88°	0,9994	0,0349	28,64
44°	0,6947	0,7193	0,9657	89°	0,9998	0,0175	57,29
45°	0,7071	0,7071	1,0000	90°	1,0000	0	

8. The following quadratic form is given: $\lambda x^2-(\lambda^2+1)x+\lambda, \quad \lambda \neq 0$

1. Find the discriminant and show that the quadratic has real roots for any $\lambda \neq 0$.

2. If x_1, x_2 are the roots of the quadratic, express the sum $S=x_1+x_2$ in terms of $\lambda \neq 0$ and find the value of the product $P=x_1 \cdot x_2$ of the roots.

3. If $\lambda > 0$, are the roots of the quadratic positive or negative? Justify your answer.

4. If $0<\lambda \neq 1$ and x_1, x_2 are the roots of the above quadratic, then compare the numbers $\frac{x_1+x_2}{2}$ and 1.

9. **1.** Choose, without justification, if the following statements are true (T) or false (F).

i. If $b \geq 0$, then $\sqrt{\alpha^2 b} = \alpha\sqrt{b}$.

ii. For any $\alpha, b \geq 0$ holds $\sqrt{a^2 + b^2} = a + b$.

iii. If $\alpha \geq 0$, we can always write $\sqrt[6]{\alpha^3} = \sqrt{\alpha}$.

2. For the following questions, choose (without justification) the correct answer

i. By the equation $|x| + |y| = 0$ can be deduced that:

a. $x > 0$ and $y > 0$
b. $|x|$ and $|y|$ are opposite numbers
c. $x = 0$ and $y = 0$
d. $x > 0$ and $y < 0$.

ii. If $x < 0$ and $y > 0$ then

a. $|x| + |y| = x + y$
b. $|x| + |y| \geq |x + y|$
c. $|x| - |y| = -x - y$
d. $|y| - |x| = |x - y|$

iii. If the equation $|2 - x| = -x + 2$ holds, then

a. $x \geq 2$
b. $x \geq 0$
c. $x \leq 2$
d. $0 \leq x \leq 2$

3. Show that the following expression is independent of $n \in \mathbb{N}$.

$$\frac{(8^{n+1} + 8^n)^2}{(4^n - 4^{n-1})^3}$$

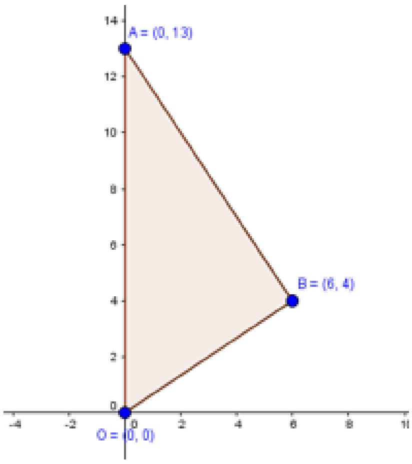
10. The diagram shows the points O(0, 0), A(0, 13) and B (6, 4).

I. Find the distance between the points A and B.

II. Find the length of the line segment OB.

III. Decide whether OAB is a right triangle. Justify your answer.

IV. Find the area of the triangle AOB.



11. The equation $2x^2 + x + c = 0$, where $c \in \mathbb{R}$, has two -distinct- real roots r_1, r_2.

I. Write the sum, S, and the product, P, of these roots.

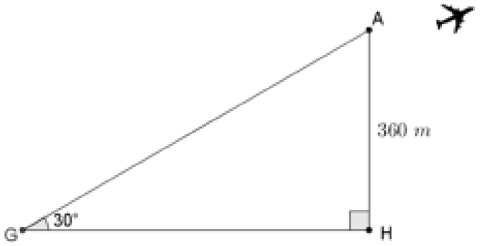
II. If it is also given that $r_2 = \frac{r_1^2}{2}$, show that $r_1 = -1$, $r_2 = \frac{1}{2}$.

III. Find the value of $c \in \mathbb{R}$.

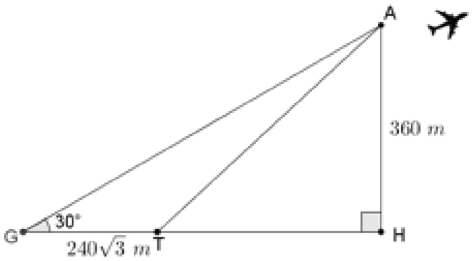
IV. Determine whether $2x^2 + x - 1$, is positive, negative or zero.

12. Gunter is at Berlin Airport watching the planes take off. He observes a plane that is at an angle of elevation of 30° of where he is standing at point G. The plane is at a height of 360 meters. This information is shown in the following diagram.

a. Calculate the horizontal distance, GH, of the plane from Gunter.



The plane took off from point T, which is $240\sqrt{3}$ meters from where Gunter is standing, as shown in the following diagram.



b. Using your answer from part (a), calculate the angle ATH, that is the take off angle of the plane.

13. A. For the following questions, choose the correct answer:

I. If $a+b=5$ and $a^2-b^2=30$, the value of $a-b$ is:

A. -5 **B.** -6 **C.** 8 **D.** 6

II. If $(a+b)^2=36$ and $a^2+b^2=68$, the value of ab is:

A. -32 **B.** 12 **C.** -16 **D.** -18

B. Simplify the following expression: $\dfrac{x^3-5x^2+4x-20}{3x^2-75} : \dfrac{x^2+4}{x^2+10x+25}$

14.

A1. a. Factorize the following expressions: $A = x^2 - 10x + 25$ and $B = 25 - x^2$.

b. Simplify the expression $\Gamma = \frac{(x^2-10x+25)^{2021}(x+5)^{2021}}{(5-x)^{2021}\,(25-x^2)^{2021}}$

A2. In the following questions, choose the correct answer

a. If $x < 2$, then $A = |2x - 4| + 3$ is simplifies to

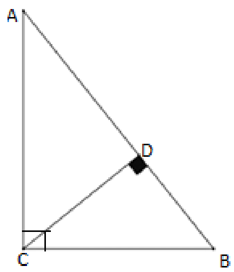
i. $A = 2x - 1$

ii. $A = 1 - 2x$

iii. $A = 2x + 7$

iv. $A = 7 - 2x$

b. In the right triangle ABC alongside holds:



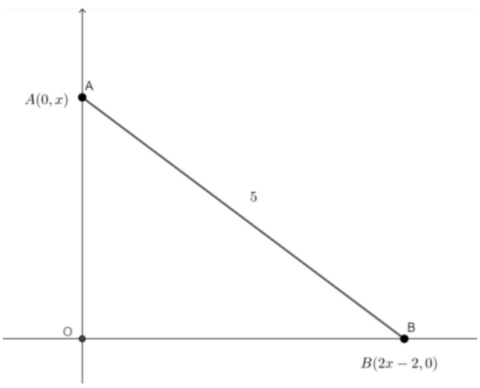
i. $sinA = \frac{\Gamma\Delta}{AB}$

ii. $sinA = \frac{A\Delta}{A\Gamma}$

iii. $cosA = \frac{A\Gamma}{AB}$

iv. $cosA = \frac{B\Gamma}{AB}$

15.



B1. Show that $5x^2 - 8x - 21 = 0$

B2. Find the area of the triangle AOB.

B3. Find the height of the triangle AOB that lies on the side AB.

B4. If the gradient of the line AB is -0.75, find the equation of this line in the form $y = mx + c$.

16.

Let $A = \frac{x^2-16}{x^2-4x}$.

C1. For which values of x is A defined?

C2. Solve the equation $|A| = 2$.

C3. Solve the inequality $A \leq 2$.

17.

Let the equation $|a-2|x^2 + |1-2a|x + |2-a| = 0$ with $a \neq 2$. The equation has two real and distinct real roots.

D1. Find the possible values of α.

D2. Show that the roots of the equation are negative and reciprocals.

D3. If the first root (r_1) is four times the other (r_2), find the two roots.

18.

A1. Write down if the following statements are True (T) or False (F) (no justification of your answer is required).

In the right triangle ABC ($\hat{C} = 90°$) the following statements hold

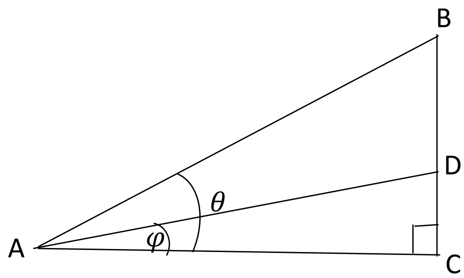
a. $tan\theta > tan\varphi$

b. $A\Gamma^2 = A\Delta^2 - \Delta\Gamma^2$

c. $cos\theta > cos\varphi$

d. $AB^2 = AD^2 + DB^2$

e. $tan(\theta - \varphi) = \frac{BD}{AD}$



A2. The expressions

$$A = 25\alpha^2 + 20\alpha b + 4b^2 \quad \text{and} \quad B = 9c^2 - 3cd + \frac{d^2}{4}$$

are given where a, b, c, d are real numbers.

a. Factorize completely the expression $A - B$.

b. Find the relationship that α, b should satisfy and c, d should satisfy such that $A + B = 0$.

19.

Juan pays 8.75 euros for a single movie ticket. The total amount Juan pays for movie tickets in a year can be modelled by $y = 8.75x$, where x represents the number of tickets purchased per year and y represents the total amount, in euros, paid per year.

Last year Juan spent less than 88 euros.

B1. Determine the maximum number of movie tickets Juan purchased last year.

Maureen buys an annual movie ticket discount card for 50 euros and then pays 2.50 euros for each movie ticket.

B2. Write down an equation in terms of x and y, using Maureen's information.

During this year, Juan and Maureen will **each** buy the same number of tickets and will each pay the same total amount of money.

B3. Find the number of tickets Juan will buy this year.

20.

The numbers $\alpha = \sqrt{(\sqrt{2}-5)^2} - \sqrt{(2-\sqrt{2})^2}$ and $b = \sqrt{2}\sqrt{2-\sqrt{2}}\sqrt{2+\sqrt{2}}$ are given.

C1. Find the values of a and of b.

C2. You are given that $a = 3$ and $b = 2$. If $\alpha < x < 2b$, show that

a. $|x-a| + |x-2b| = 1$

b. $x^3 - 2bx^2 < 3ax - 6ab$

21.

The quadratic $4x^2 - 4\lambda x + (4\lambda - 3)$, is given where λ is a real number.

D1. Show that the discriminant of the quadratic is $\Delta = 16(\lambda - 1)(\lambda - 3)$.

D2. Find the possoble values of λ such that the quadrtatic has two distinct real roots.

D3. For $\lambda = 2.9$, solve the inequality $4x^2 - 4\lambda x + (4\lambda - 3) < 0$.

D4. If x_1, x_2 are the roots of the quadratic, find the value of λ given that $S + P = \frac{29}{4}$ (where S is the sum and P is the product of the roots of the quadratic).

D5. For $\lambda = 4$, find the value of the expression $|x_1 + 1||x_2 + 1|$, without finding the roots of the quadratic.

22.

A1. Write down if the following statements are True (T) or False (F) (no justification of your answer is required).

f. If $\alpha^2 + \beta^2 = 0$ is true for any real numbers α and b, then $\alpha = 0$ or $\beta = 0$.

g. If $\alpha < 0$, then $|\alpha| < 0$.

h. If $|x| \leq 5$, then $x \leq 5$ or $x \leq -5$.

i. For any number $x \geq 0$ holds that $\sqrt{(-x)^2} = x$.

j. For any real numbers α, β holds that $(-\alpha - \beta)^2 = (\alpha + \beta)^2$

A2. Choose the correct answer

a. If $|x| > 5$ then

A. $x = 5$ Γ. $x \in (-5,5)$
B. $x = -5$ Δ. $x \in (-\infty, -5) \cup (5, +\infty)$

b. The expression $\sqrt{5} + \sqrt{5}$ is equal to

A. $\sqrt{10}$ Γ. $\sqrt{5}$
B. 5 Δ. $\sqrt{20}$

c. If $3 < x < 4$, then the expression $\frac{|x-4|}{x-4} + \frac{|x-3|}{x-3}$ is equal to

A. 2 Γ. 0

B. 0 Δ. -2

d. The expression $\sqrt{\sqrt{2} - 1}\ \sqrt{1 + \sqrt{2}}\ \sqrt{2}$ is equal to

A. $\sqrt{2}$ Γ. $\sqrt{8}$

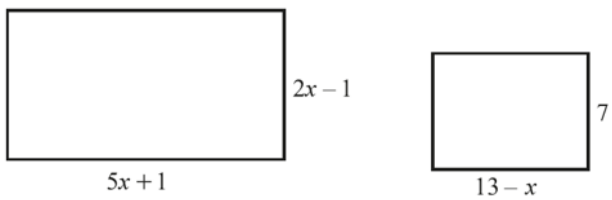
B. 1 Δ. 2

A3. Factorize completely the following expression:

$$\frac{5x - 2x^2 - 3}{6x^3 - 6x}$$

23.

(In this question all lengths are in cm. It is given that $\sqrt{1764} = 42$)



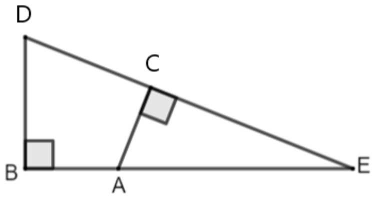
The area of the larger rectangle is 84cm^2 greater than the area of the smaller rectangle.

B1. Show that $5x^2 + 2x - 88 = 0$.

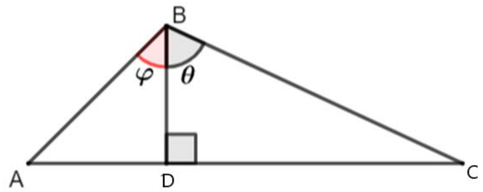
B2. Find the area of the smaller rectangle.

24.

C1. In the following diagram $EA = 10cm, EB = 12cm$ and $EC = 8cm$. Find the lengths ED, AC and BD



C2. In the following triangle ABC it is given that $A\Gamma = 9cm$, $\tan\theta = 1{,}2$ και $\tan\phi = 1{,}8$. Find the length of the height BD.



25.

The equation $(8-\lambda)x^2 - 2(\lambda - 2)x + 1 = 0$ is given where λ is a real number.

D1. Solve the equation for $\lambda = 8$.

D2. For $\lambda \neq 8$, show that the discriminant is $\Delta = 4\lambda^2 - 12\lambda - 16$.

D3. For which values of λ the equation has no solution?

D4. For the values of λ that the equation has two distinct real roots x_1, x_2, show that

$$x_1 + x_2 + 12x_1x_2 = 1 + 3\,\lambda x_1x_2$$

26.

A1. Write down if the following statements are True (T) or False (F) (no justification of your answer is required).

k. $d(x, -3) = |x + 3|$ is true that for any real number x.

l. $|x - y| \leq |x| - |y|$ is true for any real numbers x, y.

m. There are no real values of x for which $\sqrt{-x}$ is defined.

n. The inequality $x^2 + \lambda x + \lambda^2 > 0$ with $\lambda \neq 0$, is true for any $x \in \mathbb{R}$.

o. The inequalities $x^2(x-1) \geq 0$ and $x - 1 \geq 0$ have the same set of solutions.

A2. Choose the correct answer.

e. The number $\sqrt{\left(9-\sqrt{7}\right)^{2}}$ is equal to

A. $9-\sqrt{7}$ Γ. $\sqrt{7}-9$
B. 2 Δ. $9+\sqrt{5}$

f. The expression $|x-2| \leq 3$ can be expressed as union of intervals in the form:

A. $x \in (-\infty,-1] \cup [5,+\infty)$ Γ. $x \in (-\infty,-2] \cup [3,+\infty)$
B. $x \in [-1,5]$ Δ. $x \in [-2,3]$

g. If $x < y < w$, then the expression $|w-y| + |y-x| - |x-w|$ is equal to

A. $2w-2x$ Γ. 0

B. $2y-2x$ Δ. $2y-2w$

h. If the inequality $x^2-2x+\gamma > 0$ is true for any real number x, then:

A. $\gamma < 1$ Γ. $\gamma = 1$

B. $\gamma \leq 1$ Δ. $\gamma > 1$

i. The equation $|x-1| = x-1$

A.Has no solutions Γ. Has unique solution $x = 1$

B. Is true for any $x < 1$ Δ. Is true for any $x \geq 1$

27.

Let α and β real numbers such that $\alpha \neq -\beta$ and

$$A = \frac{\alpha(\alpha-1)^2 + \beta^2\alpha + \beta(1-\alpha)^2 + \beta^3}{\alpha+\beta}$$

B1. Show that A is simplified as $A = \alpha^2 + \beta^2 - 2\alpha + 1$.

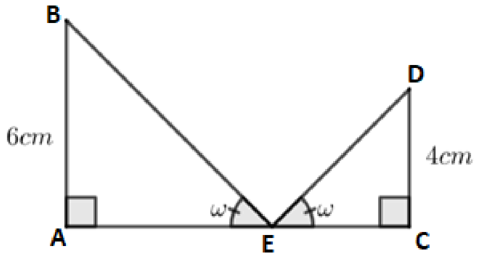
B2. Show that $A \geq 0$ for any real number α, β. When does the equation hold?

B3. The above α and β correspond to the width and the length of a rectangle respectively that satisfy the inequalities $2 < \alpha < 3$ and $3 < \beta < 5$. If we reduce the width by 1 unit, find the possible values of A.

28.

In the following diagram $AB = 6cm$, $CD = 4cm$ and $AC = 10cm$.

C1. Find the lengths of AE and EC.
C2. Find the length of the line segment BD.



29.

The equation $x^2 - 6x + k = 0$, has roots x_1 and x_2 for which

$$2x_1 + 5x_2 = 18$$

D1. Find the value of the sum of the roots S and express their product P in therms of k.

D2. Show that $x_2 = 2$.

D3. Find the value of the root x_1 and of k.

D4. For $k = 8$ and x_1, x_2 the roots of the first equation, find the possible values of the parameter μ such that the inequality

$$(\mu - 4)x^2 - \frac{2x_1 + 5x_2}{3}x + \mu < -\frac{k}{2}$$

is true for any $x \in \mathbb{R}$.